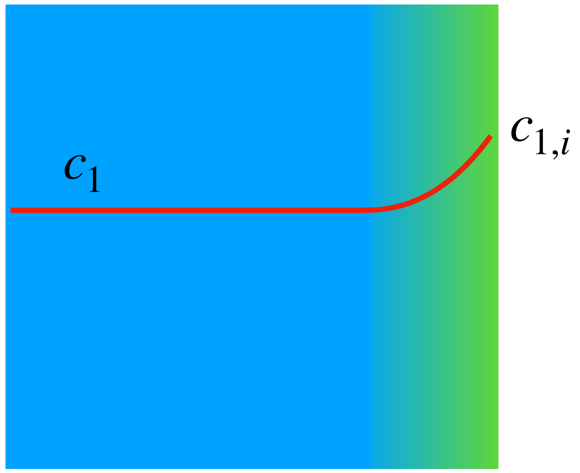
ChE-402: Diffusion and Mass Transfer

Lecture 7

Intended Learning Outcome

- Compare the concept of diffusion and mass transfer.
- Analysis of a few mass transfer cases using mass transfer coefficient.
- Derive overall mass transfer coefficient based on interfacial equilibrium.
- Analyze dimensional analysis based mass transfer correlations.
- Understanding mass transfer correlations to calculate mass transfer coefficient.

The concept of mass transfer coefficient



<u>Mass transfer coefficient is empirical but is convenient</u>
(knowledge of bulk concentrations may be sufficient)

$$\text{Flux} = \text{Mass transfer coefficient} \; * \; (\Delta \text{c})$$

$$N_1 = k(c_{1,i} - c_1)$$

<u>Diffusion coefficient is fundamental but requires knowledge of concentration gradient</u>
(e.g. we need to know precisely the thickness of film)

$$\text{Flux} = \text{Diffusion coefficient} \; * \; (-\nabla \text{c})$$

$$J_1 = -D \nabla c_1$$

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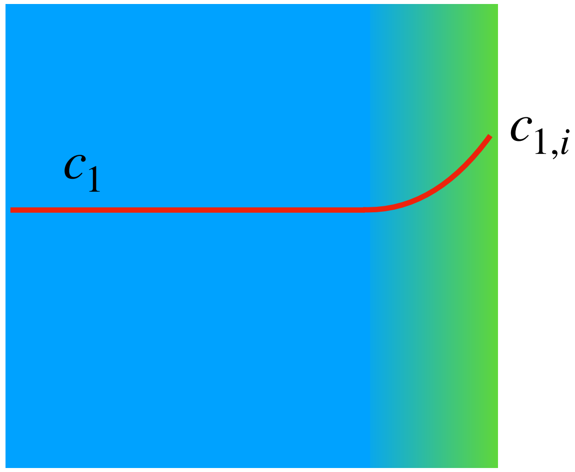
Compare the unit of k and D



$$N_1 = k(c_{1,i} - c_1)$$

Units of k = m/s

$$J_1 = -D\nabla c_1$$

Units of D = m^2/s

- Mass transfer coefficient is similar to the rate constant of reaction, written per area, not per volume.
- Because its unit is same as that of velocity, *k* is sometime referred to as ‘velocity of diffusion’.

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When should we use mass transfer coefficient and when should we use diffusion coefficient

k	D
Engineering approximation	Fundamental transport property
Complex geometry	Geometry not too complex (knowledge of length)
Well mixed bulk concentrations	Bulk is not well-mixed (transient slab problem)
When we are interested in carrying out quick/approximate estimates of mass transport	When we are interested in understanding relative contribution from diffusion and convection

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Mass transfer coefficient can evolve with time

Consider diffusion across a semi-infinite slab

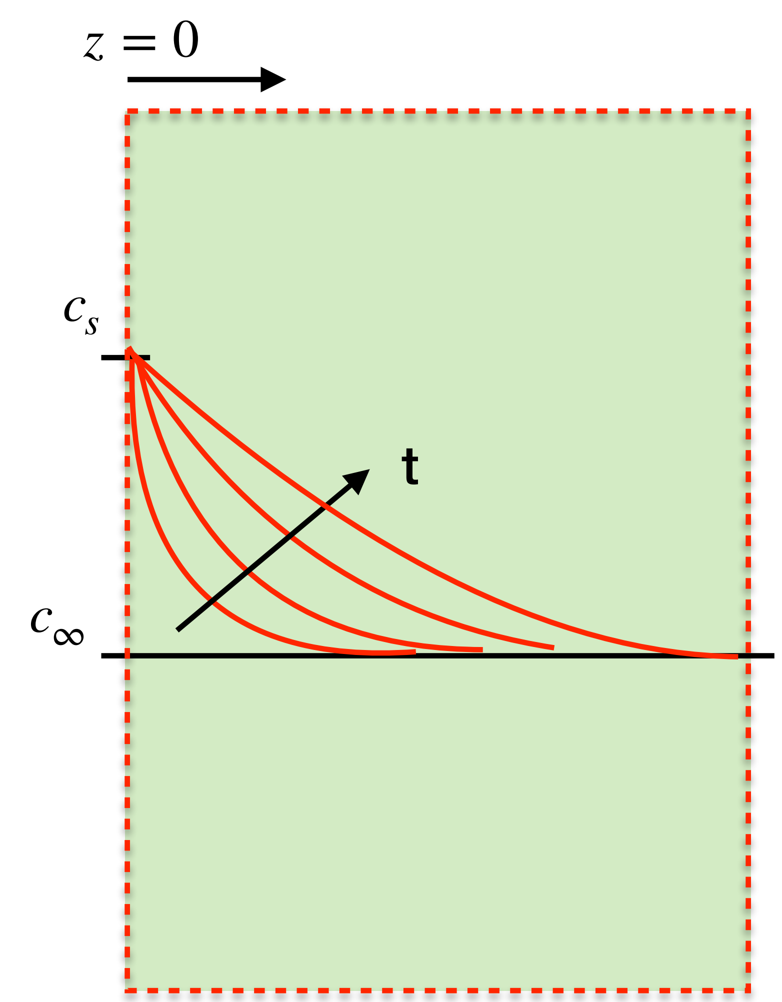
$$J_{surface} = \sqrt{\frac{D}{\pi t}}(c_s - c_\infty) = k(c_s - c_\infty)$$

$$\Rightarrow k = \sqrt{\frac{D}{\pi t}}$$

- k is initially (t =0) quite large, but reduces with time.
- If you have to use k, a time averaged k should be used.

$$\bar{k} = \frac{\int_0^\tau k dt}{\tau}$$

$$\Rightarrow \bar{k} = 2\sqrt{\frac{D}{\pi \tau}} = 2k(\tau)$$

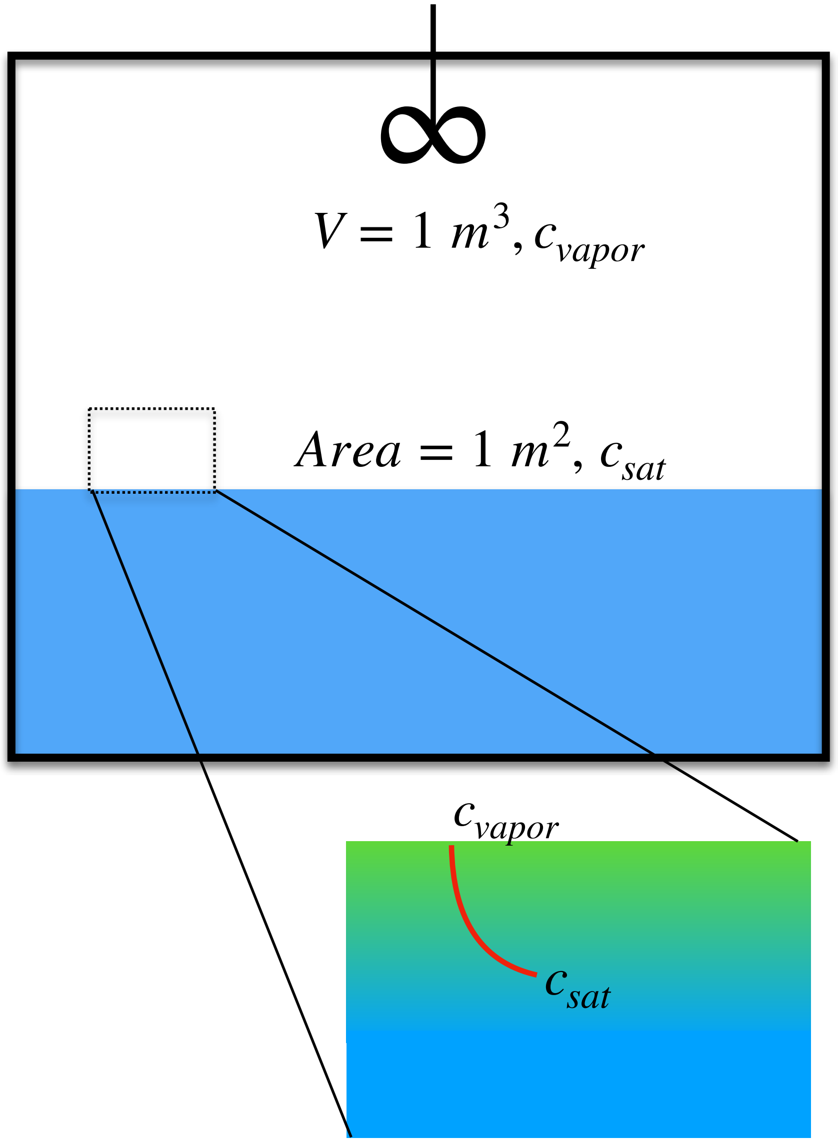


Humidification problem (liquid-vapor interface)

Water is evaporating from a bath into initially dry, well-mixed air at 1 bar. After 3 minutes, the water vapor in air is 0.003 bar. The system is isothermal at 25 ºC. Saturation pressure for water is 0.03 bar.

Calculate k for transport of water vapor to air.
Also calculate time to reach 90% saturation.

$$N_1 = k(c_{1,i} - c_1)$$



Vapor accumulation in air = Mass transferred from liquid water

$$\frac{d(Vc_{vapor})}{dt} = N_{vapor} * Area = \frac{D}{l} A(c_{sat} - c_{vapor}) = kA(c_{sat} - c_{vapor})$$

$$\Rightarrow \frac{dc_{vapor}}{dt} = \frac{kA}{V}(c_{sat} - c_{vapor})$$

Initial condition

$$t = 0;\ c_{vapor} = 0$$

$$c_{vapor} = c_{sat}(1 - e^{\frac{-kA}{V}t})$$

$$kt = \frac{V}{A} \ln\left(\frac{c_{sat}}{c_{sat} - c_{vapor}}\right)$$

$$k = 5.9 * 10^{-4}\ \text{m/s}$$

To reach 90% saturation

$$t = 3930\ \text{s}$$

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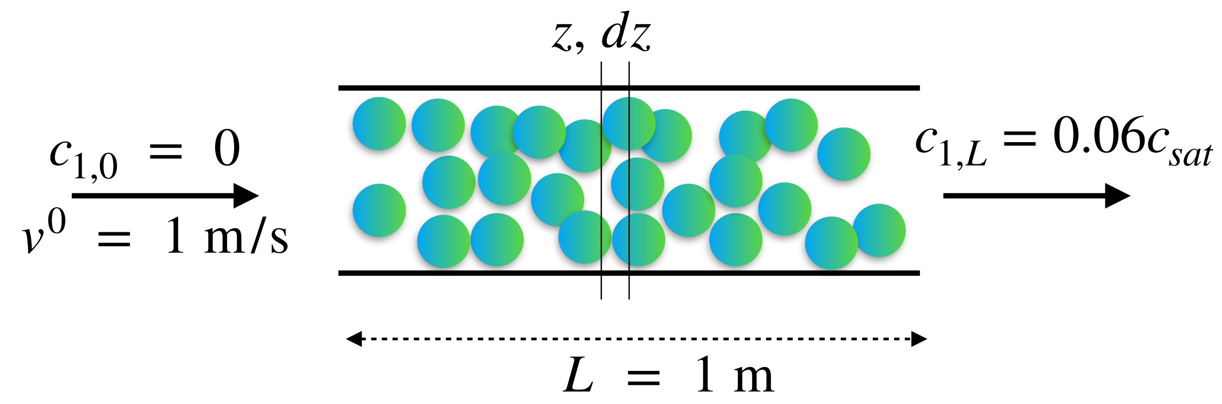
Packed bed problem (solid-liquid interface)

Pure water is flowing through a packed bed (1 m in length) made of benzoic acid beads to dissolve benzoic acid. The beads are 2 mm in diameter and have a mass transfer area of 20 cm² per cm³ volume of bed). If water comes out with a concentration of 6% saturation in steady-state conditions, calculate k for transport of benzoic acid to water.

a = **surface area per unit volume for mass exchange**

$$= \frac{20\ cm^2}{1\ cm^3} = 20\ cm^{-1}$$



How would you solve this problem?

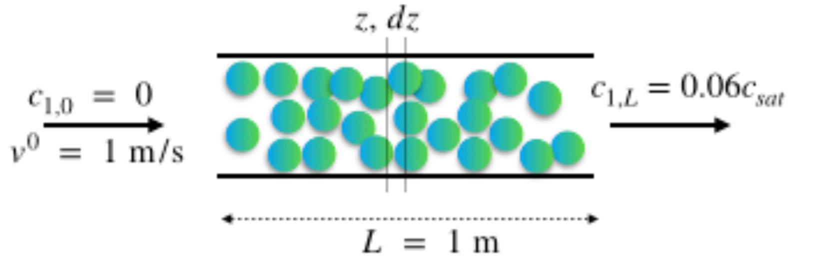
Active exchange area A = aV **where V is the volume of cylinder**

We can do mass balance at length z in an element of width dz, dV = Adz

Packed bed problem

$$\text{Accumulation in water} \;=\; (\text{flow in} \;-\; \text{flow out}) \;+\; (\text{mass transferred})$$

Steady-state, accumulation = 0



$$0 \;=\; (c_1 v^0 A\,|_z - \; c_1 v^0 A\,|_{z+dz}\,) \;+\; kaAdz(c_{1,sat} - c_1)$$

$$\Rightarrow 0 \;=\; \frac{(c_1 v^0\,|_z - \; c_1 v^0\,|_{z+dz}\,)}{dz} \;+\; ka(c_{1,sat} - c_1)$$

$$\Rightarrow -\frac{d}{dz}(c_1 v^0) \;+\; ka(c_{1,sat} - c_1) \;=\; 0$$

Dilute regime, constant v⁰

$$\Rightarrow \frac{dc_1}{dz} \;=\; \frac{ka}{v^0}(c_{1,sat} - c_1)$$

Familiar form (similar to previous problem)

$$c_1 \;=\; c_{1,sat}(1 - e^{\frac{-ka}{v^0}z})$$

$$k \;=\; \frac{v^0}{az}\ln\left(\frac{c_{1,sat}}{c_{1,sat} - c_1}\right)$$

$$k = 3.1 * 10^{-5}\ \text{m/s}$$

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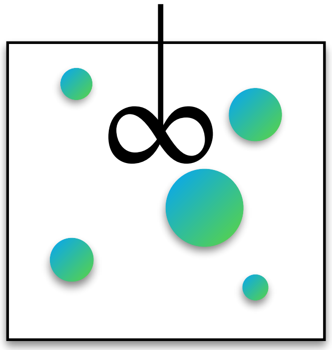
Emulsion problem (liquid-liquid interface)

Liquid bromine is rapidly dissolved in water by emulsifying liquid bromine droplets with water (mass exchange of bromine from droplet to bulk water). In 3 minutes, the concentration of bromine in water reaches 50% of saturation. Calculate ka where a is area of droplet per unit volume. Assume volume of water is constant, and a constant droplet size.



$$\text{Accumulation in water} = \text{mass transferred}$$

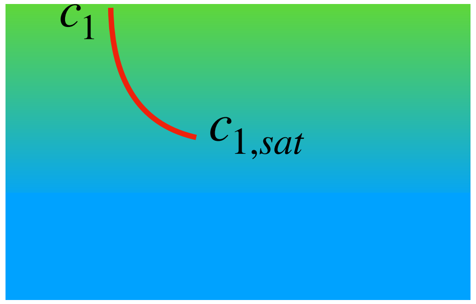
$$\frac{d}{dt}(c_1 V) = kA(c_{1,i} - c_1) = kA(c_{1,sat} - c_1)$$

$$\frac{dc_1}{dt} = k\frac{A}{V}(c_{1,sat} - c_1) = ka(c_{1,sat} - c_1)$$

a = area of the droplets per unit volume

$$c_1 = c_{1,sat}(1 - e^{-kat})$$

Again, similar form



$$ka = \frac{1}{t}\ln\left(\frac{c_{1,sat}}{c_{1,sat} - c_1}\right)$$

$$ka = 3.85 * 10^{-3} \ \text{s}^{-1}$$

Usually ka is reported.

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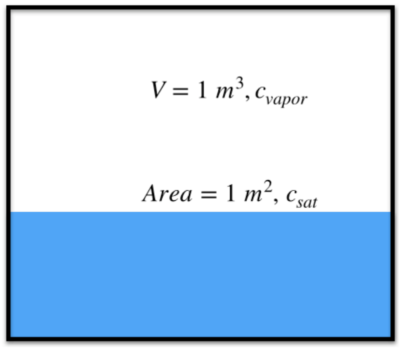
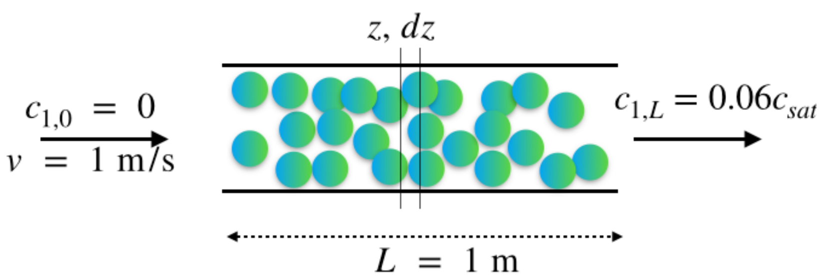
Analysis of previous 3 problems



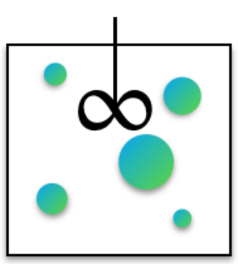
Liquid-vapor interface

$$c_{vapor} = c_{sat}(1 - e^{\frac{-kA}{V}t})$$



Solid-liquid interface

$$c_1 = c_{1,sat}(1 - e^{\frac{-ka}{v}z})$$



Liquid-liquid interface

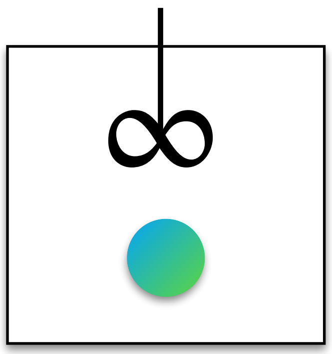
$$c_1 = c_{sat}(1 - e^{-kat})$$

Dissolving gas bubbles (gas-liquid interface)

A 2 cm diameter oxygen bubble is injected in a well-mixed infinitely large water reservoir. If bubble shrinks by 50% of its diameter in 10 minutes, calculate k for transport of oxygen in water. Saturation concentration of dissolved oxygen is 0.0015 M.

How much will the bubble shrink in 20 minutes?

Analysis on the bubble



$$\text{Mass loss of bubble} \;=\; \text{mass transferred}$$

$$\frac{d}{dt}(c_1 V) \;= -k_g A(c_1 - c_{1,sat,g}) \;= -k_l A(c_{1,sat,l} - 0)$$

$c_{1,sat,g}$ is not known

c_1 is constant

$$\Rightarrow \frac{dV}{dt} \;= -k_l A\left(\frac{c_{1,sat,l}}{c_1}\right)$$

$$c_1 = \frac{P}{RT} = \frac{10^5}{8.314 * 298} \;= 40.4 \text{ mole/m}^3$$

$$\Rightarrow \frac{d(\frac{4}{3}\pi r^3)}{dt} \;= -k_l(4\pi r^2)\left(\frac{c_{1,sat,l}}{c_1}\right)$$

$$c_{1,sat,l} = 0.0015 \text{ M} \;=\; 1.5 \text{ mole/m}^3$$

$$\Rightarrow \frac{dr}{dt} \;= -k_l \frac{c_{1,sat,l}}{c_1}$$

Integrate with initial condition, t= 0, r= r_0

$$\Rightarrow r \;=\; r_o - k_l \frac{c_{1,sat,l}}{c_1} t$$

$$k_l \;=\; 2.2 * 10^{-4} \;\; \text{ms}^{-1}$$

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Other representation of mass transfer coefficient

Representation	Units of mass transfer coefficient
$N_1 = k\Delta c_1$	$\mathrm{m\ s^{-1}}$
$N_1 = k_x \Delta x_1$	$\mathrm{mol\ m^{-2}\ s^{-1}}$
$N_1 = k_p \Delta P_1$	$\mathrm{mol\ m^{-2}\ Pa^{-1}\ s^{-1}}$

$$N_1 = k\Delta c_1 = kc\Delta x_1 = k_x \Delta x_1$$

$$\mathbf{k_x} = \mathbf{kc} \qquad \mathrm{m\ s^{-1}\ mole\ m^{-3}}$$

$$N_1 = k\Delta c_1 = k\frac{1}{RT}\Delta P_1 = k_p \Delta P_1$$

$$\mathbf{k_p} = \frac{\mathbf{k}}{\mathbf{RT}} \qquad \frac{\mathrm{ms^{-1}}}{\mathrm{Pa\ m^3\ mole^{-1}}}$$

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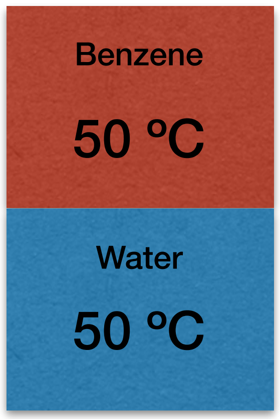
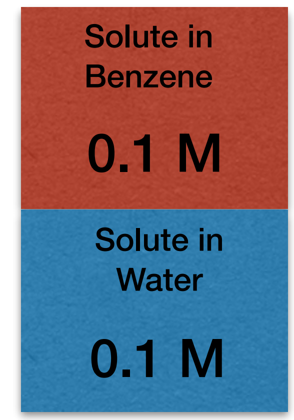
Is heat transfer possible



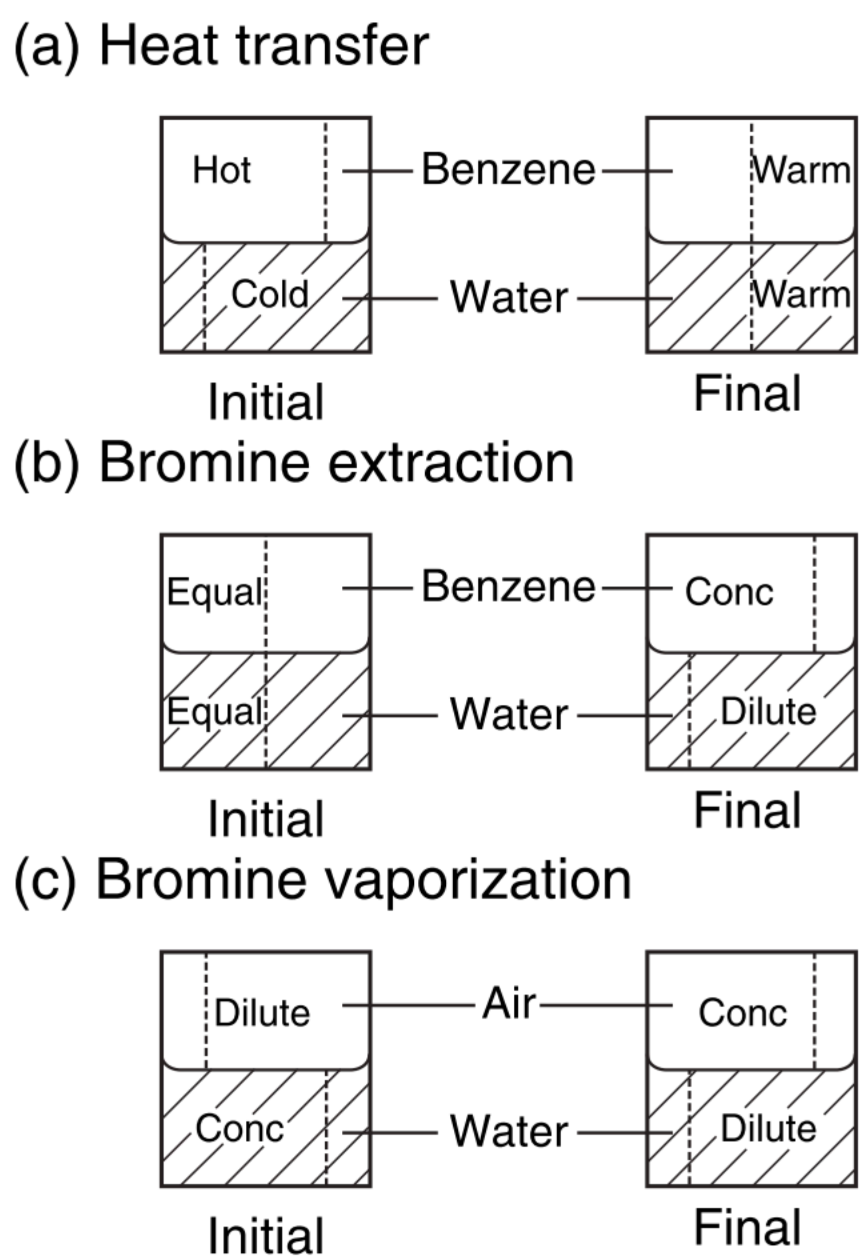
A. Yes
B. No

Is mass transfer possible (yes/no?)



A. Yes
B. No

The driving force for heat transfer is temperature but for mass transfer is chemical potential.



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For a two-phase interface, how many fundamental mass transfer coefficient exist?

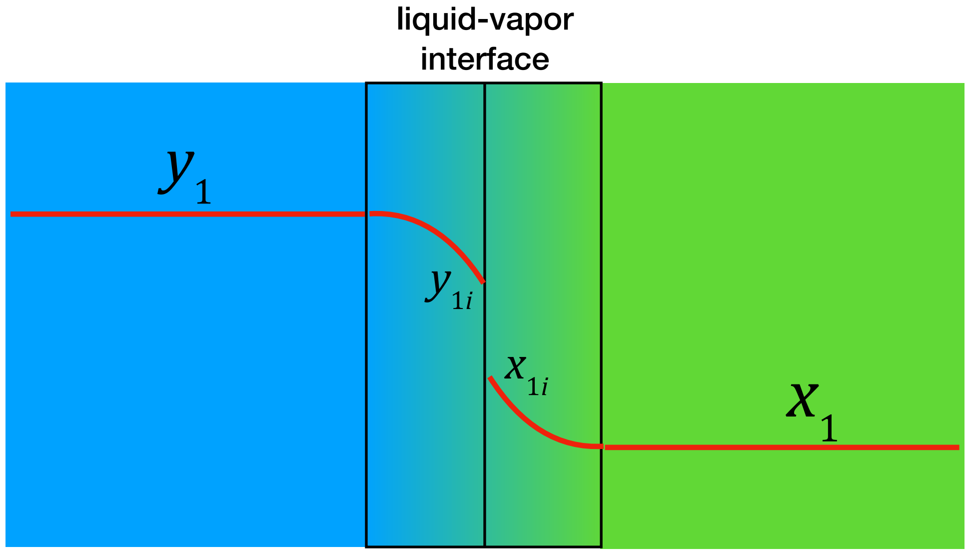
A. One

B. Two

C. Three

D. Infinite possibilities

Overall mass transfer coefficient between two phases in equilibrium



$$N_1 = k_y(y_1 - y_{1i}) \qquad N_1 = k_x(x_{1i} - x_1)$$

$$y_{1i} = m x_{1i}$$

Usually y_{1i} and x_{1i} are not known

$$\mathbf{N_1 = k_y(y_1 - y_{1i})} \qquad \mathbf{N_1 = k_x(x_{1i} - x_1)}$$

$$y_1 - y_{1i} = \frac{N_1}{k_y} \qquad x_{1i} - x_1 = \frac{N_1}{k_x}$$

$$y_1 - m x_{1i} = \frac{N_1}{k_y} \qquad m x_{1i} - m x_1 = \frac{m N_1}{k_x}$$

Add above

$$y_1 - \cancel{m x_{1i}} + \cancel{m x_{1i}} - m x_1 = \frac{N_1}{k_y} + \frac{m N_1}{k_x}$$

$$\Rightarrow y_1 - m x_1 = N_1\left(\frac{1}{k_y} + \frac{m}{k_x}\right)$$

$$\Rightarrow y_1 - m x_1 = N_1 * \frac{1}{K_y}$$

$$\Rightarrow N_1 = K_y(y_1 - m x_1)$$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

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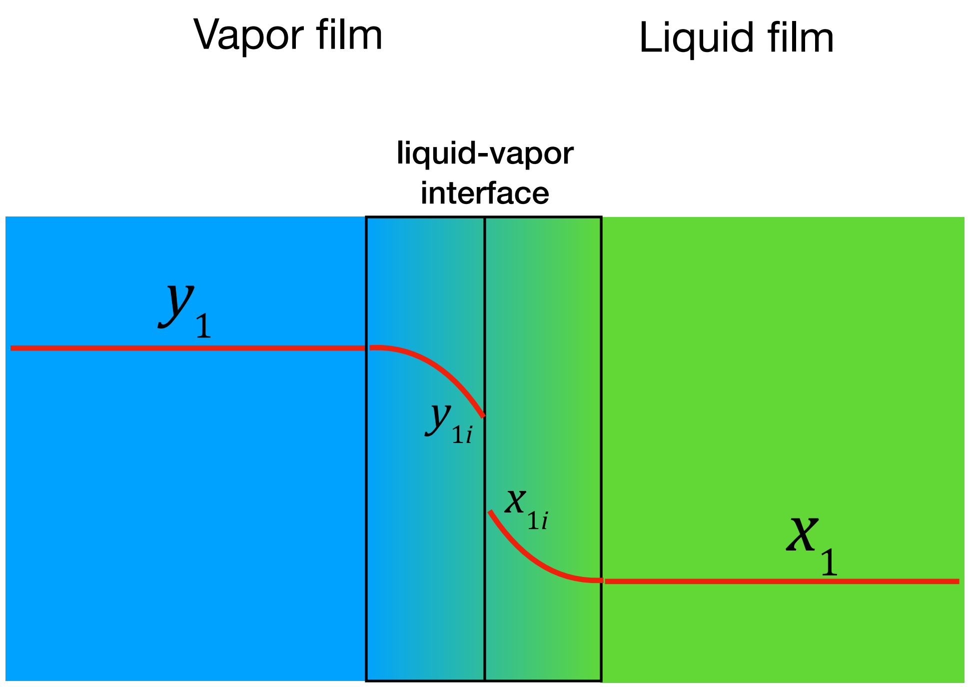
Overall mass transfer coefficient between two phases in equilibrium



$N_1 = k_y(y_1 - y_{1i})$

$N_1 = k_x(x_{1i} - x_1)$

$y_{1i} = m x_{1i}$

Usually y_{1i} and x_{1i} are not known

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

Prove

$$\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

What would be overall mass transfer coefficient from gas to liquid phase

Gas phase $N_1 = k_p(P_1 - P_{1i})$

$$N_1 = K_p(P_1 - Hx_1) = K_x\left(\frac{P_1}{H} - x_1\right)$$

Liquid phase $N_1 = k_x(x_{1i} - x_1)$

$$\frac{1}{K_p} = \frac{1}{k_p} + \frac{H}{k_x}$$

Equilibrium $H = \frac{P_{1i}}{x_{1i}}$

$$\frac{1}{K_x} = \frac{1}{Hk_p} + \frac{1}{k_x}$$

Few observations about overall coefficient

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

1. Total resistance = Phase I resistance + Phase II resistance
2. Higher resistance will dominate the overall transport.

$$k_y = 1$$

$$k_x = 10^{-4}$$

$$m = 1$$

$$K_y = 10^{-4}$$

Liquid phase (slow k) dominates

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Other notations

Sometime, you often come across

$N = K_p(P_1 - P_1^*)$ Where P_1^* is hypothetical pressure in equilibrium with bulk liquid

$N = K_x(x_1^* - x_1)$ Where x_1^* is hypothetical mole fraction in equilibrium with bulk gas

$$P_1^* = Hx_1$$

$$x_1^* = \frac{P_1}{H}$$

Note, this is not equilibrium relationship for interfacial concentration

But merely a hypothetical concentration for the sake of convenience.

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Why we need correlation for mass transfer coefficient

- Mass transfer coefficients, *k*, do not have molecular basis as that of diffusion coefficient, *D*.
- For example *k* (oxygen dissolving in water) will differ based on stirring, whereas *D* will not change.

<u>The relevance of mass transfer coefficient in the chemical industry (process scale-up)</u>

Small-scale (10 L) bioreactor

Laboratory studies

Large-scale (10000 L) bioreactor

Manufacturing

Dimensionless analysis

There are several industrially important problems of interest:

1) Fluid flow through a packed bed of particles (ion-exchange, adsorption, absorption, catalysis...).
2) Gas bubbles rising in a liquid tank (humidification, absorption, oxygenation).
3) Falling films (humidification).

- Typically, several experiments are done in a small prototype (at small scale) to correlate parameters for dimensionless correlations or validate if existing.
- Dimensionless correlations are applied for scale-up.
- The error in such correlation can be as low as 1%, and as high as 30% depending on correlation accuracy and type of interface (fluid-fluid vs fluid-solid).
- This method has been heavily inspired by heat transport analysis which has been carried out for a longer period of history for fluid-solid interfaces.

$$\frac{kl}{D} = (\text{constant})\ (\text{Re})^{\text{x}}(\text{Sc})^{\text{y}}$$

Variables important to dimensionless analysis

D = mass diffusivity =

α = thermal diffusivity = $\frac{\text{heat conducted}}{\text{heat stored}}$

ν = momentum diffusivity =

Schmidt number = $\frac{\nu}{\text{D}}$

$\frac{\text{momentum}}{\text{mass}}$

Lewis number = $\frac{\alpha}{\text{D}}$

$\frac{\text{heat}}{\text{mass}}$

Prandtl number = $\frac{\nu}{\alpha}$

$\frac{\text{momentum}}{\text{heat}}$

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What other variables that come to your mind

k = mass transfer coefficient = velocity of diffusion =

v = velocity =

D = mass diffusivity =

Length-scale (thickness, diameter) matters

$$\text{Stanton number} = \frac{k}{v}$$

$$\frac{\text{velocity of diffusion}}{\text{velocity}}$$

$$\text{Sherwood number} = \frac{kl}{D}$$

$$\frac{\text{mass transfer velocity}}{\text{mass diffusivity}}$$

$$\text{Peclet number} = \frac{vl}{D}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

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Dimensional numbers important to mass transfer

Group[a]	Physical meaning	Used in
Sherwood number $\frac{kl}{D}$	$\frac{\text{mass transfer velocity}}{\text{diffusion velocity}}$	Usual dependent variable
Stanton number $\frac{k}{v^0}$	$\frac{\text{mass transfer velocity}}{\text{flow velocity}}$	Occasional dependent variable
Schmidt number $\frac{\nu}{D}$	$\frac{\text{diffusivity of momentum}}{\text{diffusivity of mass}}$	Correlations of gas or liquid data
Lewis number $\frac{\alpha}{D}$	$\frac{\text{diffusivity of energy}}{\text{diffusivity of mass}}$	Simultaneous heat and mass transfer

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Dimensional numbers important to mass transfer

Group[a]	Physical meaning	Used in
Reynolds number $\frac{lv}{\nu}$	$\frac{\text{inertial forces}}{\text{viscous forces}}$ or $\frac{\text{flow velocity}}{\text{"momentum velocity"}}$	Forced convection
Grashof number $\frac{l^3 g \Delta\rho / \rho}{\nu^2}$	$\frac{\text{buoyancy forces}}{\text{viscous forces}}$	Free convection
Péclet number $\frac{v^0 l}{D}$	$\frac{\text{flow velocity}}{\text{diffusion velocity}}$	Correlations of gas or liquid data
Second Damköhler number or (Thiele modulus)^2 $\frac{\kappa l^2}{D}$	$\frac{\text{reaction velocity}}{\text{diffusion velocity}}$	Correlations involving reactions (see Chapters 16–17)

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Frequently used mass transfer correlations

Divided into 2 categories

Fluid-fluid interface

- Distillation
- Absorption
- Extraction
- Water aeration
- Oxygenation

Frequently used in chemical industry.
No parallel in heat transfer.

Error of the order of 30%

Fluid-solid interface

- Membranes
- Adsorption
- Leaching
- Catalyst bed

Heavily inspired from heat transfer.

Error at around 10%,
At best 1% (laminar flow in a single tube)

Laminar flow of one fluid in a tube is much better understood than turbulent flow of gas and liquid in a packed tower

- **Due to the errors, these correlations are often used to design a small-scale (pilot plant).**
- **Typically you will validate the result in pilot plant with the actual chemicals, and then make a scale-up model.**

Fluid-fluid interface are difficult to handle

- *Re* can vary by a factor of 10000 (laminar vs. turbulent flow). $Re = \frac{dv\rho}{\eta}$

- Correlation involve velocities. Velocities have 2 origins.
  - Forced (stirring, pumping).
  - Free (density gradients, gravity).

- Sc in gases ~ 1, in liquids ~ 1000.

Water at 25 ºC as an example:

Liquid:	$\eta = 0.001$ Pa s	$\rho = 1000$ Kg/m^3	$D = 10^{-9}$ m^2/s	$Sc =$
Vapor:	$\eta = 10^{-5}$ Pa s	$\rho = 1$ Kg/m^3	$D = 10^{-5}$ m^2/s	$Sc =$

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Water vapor viscosity from
Journal of Chemical and Engineering Data, Vol. 50, No. 6, 2005

How dimensionless correlations are made

$$ka = ka(v, \rho, \mu, d, z)$$

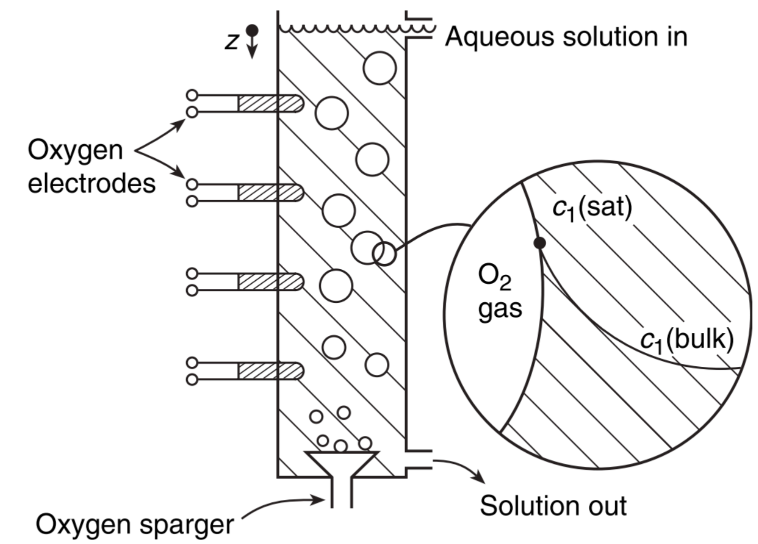
Assume the function has an exponential form

$$ka = [\text{constant}]\mathrm{v}^{\alpha}\rho^{\beta}\mu^{\gamma}\mathrm{d}^{\delta}\mathrm{z}^{\epsilon}$$

Check dimension on both sides

$$\frac{L}{t}\frac{1}{L} \; [=] \; \left(\frac{L}{t}\right)^{\alpha}\left(\frac{M}{L^3}\right)^{\beta}\left(\frac{M}{Lt}\right)^{\gamma} L^{\delta}\, L^{\epsilon}$$

$$t^{-1} \;\; [=] \;\; L^{(\alpha-3\beta-\gamma+\delta+\epsilon)} \;\; t^{(-\alpha-\gamma)} \;\; M^{(\beta+\gamma)}$$



The only way this equation can be dimensionally consistent is if the exponent on time on the left-hand side of the equation equals the sum of the exponents on time on the right- hand side

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How dimensionless correlations are made

$$t^{-1} \; [=] \; L^{(\alpha-3\beta-\gamma+\delta+\epsilon)} \; t^{(-\alpha-\gamma)} \; M^{(\beta+\gamma)}$$

$$\Rightarrow \alpha - 3\beta - \gamma + \delta + \epsilon \; = \; 0$$

$$\Rightarrow -\alpha - \gamma \; = -1$$

$$\Rightarrow \beta + \gamma \; = \; 0$$

3 equations, 5 variables

At least 2 dependent variables

If we arbitrarily choose γ and ϵ as the dependent variable

$$\alpha = 1 - \gamma$$

$$\beta = -\gamma$$

$$\gamma = \gamma$$

$$\delta = -\gamma - \varepsilon - 1$$

$$\varepsilon = \varepsilon$$

$$ka = [\text{constant}]\text{v}^{\alpha}\rho^{\beta}\mu^{\gamma}\text{d}^{\delta}\text{z}^{\epsilon}$$

$$\Rightarrow ka = [\text{constant}]\text{v}^{(1-\gamma)}\rho^{(-\gamma)}\mu^{\gamma}\text{d}^{(-\gamma-\epsilon-1)}\text{z}^{\epsilon}$$

$$\Rightarrow ka = [\text{constant}]\frac{\text{v}}{\text{v}^{\gamma}}\frac{1}{\rho^{\gamma}}\mu^{\gamma}\frac{1}{\text{d}^{\gamma}}\;\frac{1}{\text{d}^{\epsilon}\text{d}}\text{z}^{\epsilon}$$

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How dimensionless correlations are made

$$\Rightarrow ka = [\text{constant}]\frac{\text{v}}{\text{v}^{\gamma}}\frac{1}{\rho^{\gamma}}\mu^{\gamma}\frac{1}{\text{d}^{\gamma}}\ \frac{1}{\text{d}^{\epsilon}\text{d}}\text{z}^{\epsilon}$$

$$\text{Stanton number} = \frac{\text{k}}{\text{v}}$$

$$\Rightarrow \frac{kad}{v} = [\text{constant}]\left(\frac{\mu}{\text{dv}\rho}\right)^{\gamma}\left(\frac{\text{z}}{\text{d}}\right)^{\epsilon}$$

$$\text{Reynolds number} = \frac{\text{dv}\rho}{\mu}$$

$$St \ = \ [\text{constant}]\ \text{Re}^{-\gamma}\left(\frac{\text{z}}{\text{d}}\right)^{\epsilon}$$

Experimental method

Plot Stanton number vs Re and z/d to get the exponents

Challenges in such analysis

- Will only work if we select the right key variable (in this case, γ and ϵ)
- Assumption that the bulk liquid is well mixed
- Data may not fit the exponential form
- We usually do not know at the start as to which variables are important (too many, too less)

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Frequently used mass transfer correlations

General patterns: correlation of ***Sh (or St)*** **to** ***Re*** **and** ***Sc***

$$\text{Sherwood number} = \frac{\text{kl}}{\text{D}} \qquad \text{Reynolds number} = \frac{\text{dv}}{\nu}$$

$$\text{Stanton number} = \frac{\text{k}}{\text{v}} \qquad \text{Schmidt number} = \frac{\nu}{\text{D}}$$

Packed beds	$\frac{k}{v^0} = 1.17\left(\frac{dv^0}{\nu}\right)^{-0.42}\left(\frac{D}{\nu}\right)^{2/3}$	d = particle diameter v^0 = superficial velocity	The superficial velocity is that which would exist without packing

Can you write above correlation in terms of Sh (or St) as a function of Sc, and Re ??

$$\text{St} = 1.17\ \text{Re}^{-0.42}\ \text{Sc}^{-2/3}$$

Fluid-fluid interface (Packed Tower)

Physical situation	Basic equation[b]	Key variables	Remarks
Liquid in a packed tower	$k\left(\frac{1}{\nu g}\right)^{1/3} = 0.0051\left(\frac{v^0}{a\nu}\right)^{0.67}\left(\frac{D}{\nu}\right)^{0.50}(ad)^{0.4}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for liquids; tends to give lower value than other correlations
	$\frac{kd}{D} = 25\left(\frac{dv^0}{\nu}\right)^{0.45}\left(\frac{\nu}{D}\right)^{0.5}$	d = nominal packing size	The classical result, widely quoted; probably less successful than above
Gas in a packed tower	$\frac{k}{aD} = 3.6\left(\frac{v^0}{a\nu}\right)^{0.70}\left(\frac{\nu}{D}\right)^{1/3}(ad)^{-2.0}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for gases
	$\frac{kd}{D} = 1.2\,(1-\varepsilon)^{0.36}\left(\frac{dv^0}{\nu}\right)^{0.64}\left(\frac{\nu}{D}\right)^{1/3}$	d = nominal packing size ε = bed void fraction	Again, the most widely quoted classical result

$k\left(\frac{1}{\nu g}\right)^{1/3}$ is unusual form of St

v^0 is superficial velocity (Velocity that exists without packing)

Can you write above correlations in terms of Sh (or St) and Re and Sc ?

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Fluid-fluid interface (Gas bubbles in tank)

Physical situation	Basic equation[b]	Key variables	Remarks
Pure gas bubbles in a stirred tank	$\frac{kd}{D} = 0.13 \left(\frac{(P/V)\, d^4}{\rho \nu^3}\right)^{1/4} \left(\frac{\nu}{D}\right)^{1/3}$	d = bubble diameter P/V = stirrer power per volume	Note that k does not depend on bubble size
Pure gas bubbles in an unstirred tank	$\frac{kd}{D} = 0.31 \left(\frac{d^3\, g\, \Delta\rho/\rho}{\nu^2}\right)^{1/3} \left(\frac{\nu}{D}\right)^{1/3}$	d = bubble diameter $\Delta\rho$ = density difference between bubble and surrounding fluid	0.3-cm diameter or larger

$\frac{d^3 g(\Delta\rho/\rho)}{\nu^2}$ is Grashof number

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Fluid-fluid interface (some others)

Physical situation	Basic equation[b]	Key variables	Remarks
Small liquid drops rising in unstirred solution	$\frac{kd}{D} = 1.13\left(\frac{dv^0}{D}\right)^{0.8}$	d = drop diameter v^0 = drop velocity	These small drops behave like rigid spheres
Falling films	$\frac{kz}{D} = 0.69\left(\frac{zv^0}{D}\right)^{0.5}$	z = position along film v^0 = average film velocity	Frequently embroidered and embellished

What dimensionless number are these ?

$$\text{Peclet number} = \frac{\text{vl}}{\text{D}}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

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Fluid-solid interface

Table 8.3-3 *Selected mass transfer correlations for fluid–solid interfaces*[a]

Physical situation	Basic equation[b]	Key variables	Remarks
Membrane	$\frac{kl}{D} = 1$	l = membrane thickness	Often applied even where membrane is hypothetical
Laminar flow along flat plate[c]	$\frac{kL}{D} = 0.646 \left(\frac{Lv^0}{\nu}\right)^{1/3} \left(\frac{\nu}{D}\right)^{1/3}$	L = plate length v^0 = bulk velocity	Solid theoretical foundation, which is unusual $Re < 2000$
Turbulent flow through horizontal slit	$\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu}\right)^{0.8} \left(\frac{\nu}{D}\right)^{1/3}$	v^0 = average velocity in slit $d = [2/\pi]$ (slit width)	Mass transfer here is identical with that in a pipe of equal wetted perimeter
Turbulent flow through circular pipe	$\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu}\right)^{0.8} \left(\frac{\nu}{D}\right)^{1/3}$	v^0 = average velocity in slit d = pipe diameter	Same as slit, because only wall regime is involved
Laminar flow through circular tube	$\frac{kd}{D} = 1.62 \left(\frac{d^2 v^0}{LD}\right)^{1/3}$	d = pipe diameter L = pipe length v^0 = average velocity in tube	Very strong theoretical and experimental basis $Re < 2000$
Flow outside and parallel to a capillary bed	$\frac{kd}{D} = 1.25 \left(\frac{d^2 v^0}{\nu l}\right)^{0.93} \left(\frac{\nu}{D}\right)^{1/3}$	d = 4 cross-sectional area/(wetted perimeter) v^0 = superficial velocity	Not reliable because of channeling in bed
Flow outside and perpendicular to a capillary bed	$\frac{kd}{D} = 0.80 \left(\frac{dv^0}{\nu}\right)^{0.47} \left(\frac{\nu}{D}\right)^{1/3}$	d = capillary diameter v^0 = velocity approaching bed	Reliable if capillaries evenly spaced
Forced convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6 \left(\frac{dv^0}{\nu}\right)^{1/2} \left(\frac{\nu}{D}\right)^{1/3}$	d = sphere diameter v^0 = velocity of sphere	Very difficult to reach $(kd/D) = 2$ experimentally; no sudden laminar-turbulent transition
Free convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6 \left(\frac{d^3 \Delta\rho g}{\rho \nu^2}\right)^{1/4} \left(\frac{\nu}{D}\right)^{1/3}$	d = sphere diameter g = gravitational acceleration	For a 1-cm sphere in water, free convection is important when $\Delta\rho = 10^{-9}$ g/cm^3
Packed beds	$\frac{k}{v^0} = 1.17 \left(\frac{dv^0}{\nu}\right)^{-0.42} \left(\frac{D}{\nu}\right)^{2/3}$	d = particle diameter v^0 = superficial velocity	The superficial velocity is that which would exist without packing
Spinning disc	$\frac{kd}{D} = 0.62 \left(\frac{d^2\omega}{\nu}\right)^{1/2} \left(\frac{\nu}{D}\right)^{1/3}$	d = disc diameter ω = disc rotation (radians/time)	Valid for Reynolds numbers between 100 and 20,000

Notes: [a] The symbols used include the following: D is the diffusion coefficient of the material being transferred; k is the local mass transfer coefficient; ρ is the fluid density; ν is the kinetmatic viscosity. Other symbols are defined for the specific situation.

[b] The dimensionless groups are defined as follows: (dv^0/ν) and $(d^2\omega/\nu)$ are the Reynolds number; ν/D is the Schmidt number; $(d^3\Delta\rho g/\rho\nu^2)$ is the Grashof number, kd/D is the Sherwood number; k/v^0 is the Stanton number.

[c] The mass transfer coefficient given here is the value averaged over the length L.

Exercise problem 1

Scrubbing problem

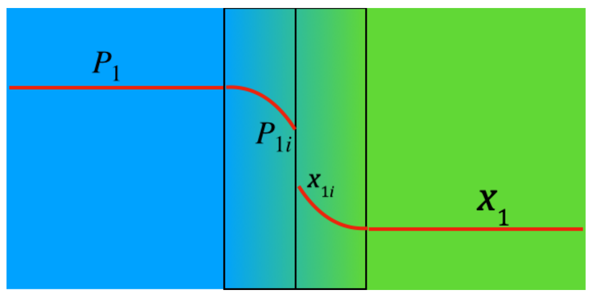
Ammonia in a carrier gas (air) is being scrubbed from the gas phase to a solvent in a packed tower. The interaction (absorption) of NH_3 with the solvent is extremely strong and irreversible. Calculate the concentration of ammonia as a function of column height.

$$x_{1i} = \bar{H} P_{1i}$$

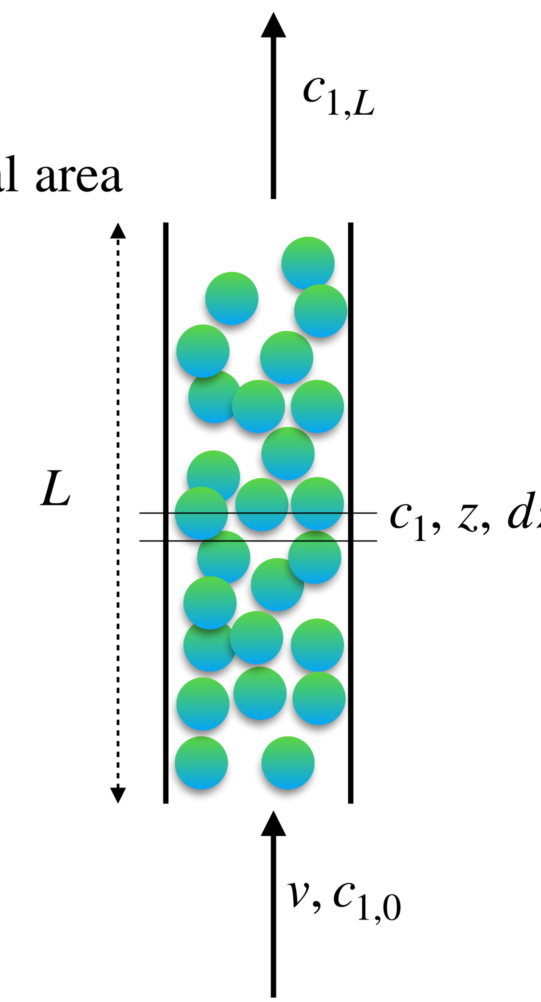
$$\Rightarrow \bar{H} \to \infty$$

$$\Rightarrow P_{1i} \to 0$$

$$\Rightarrow c_{1i} \to 0 \text{ (in gas phase)}$$



A = cross − sectional area



Analysis on gas phase at height z

Accumulation in gas phase = (flow in − flow out) − (mass lost to absorption)

Steady-state, accumulation = 0

$$0 = (c_1 v) * A_1 |_z - (c_1 v) * A_1 |_{z+dz} - k A_2 (c_1 - c_{1,i})$$

$$A_1 = A$$

$$\Rightarrow (c_1 v) * A |_z - (c_1 v) * A |_{z+dz} - kaAdz(c_1 - 0)$$

$$A_2 = aAdz$$

$$\Rightarrow \frac{dc_1}{dz} = -\frac{kac_1}{v}$$

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Ammonia in a carrier gas (air) is being scrubbed from the gas phase to a solvent in a packed tower. The interaction (absorption) of NH_3 with the solvent is extremely strong and irreversible. Calculate the concentration of ammonia as a function of column height.

$$\Rightarrow \frac{dc_1}{dz} = -\frac{kac_1}{v}$$

$$\Rightarrow \ln c_1 = -\frac{kaz}{v} + \text{constant}$$

$c_1 = c_{1,0}$ when z = 0,

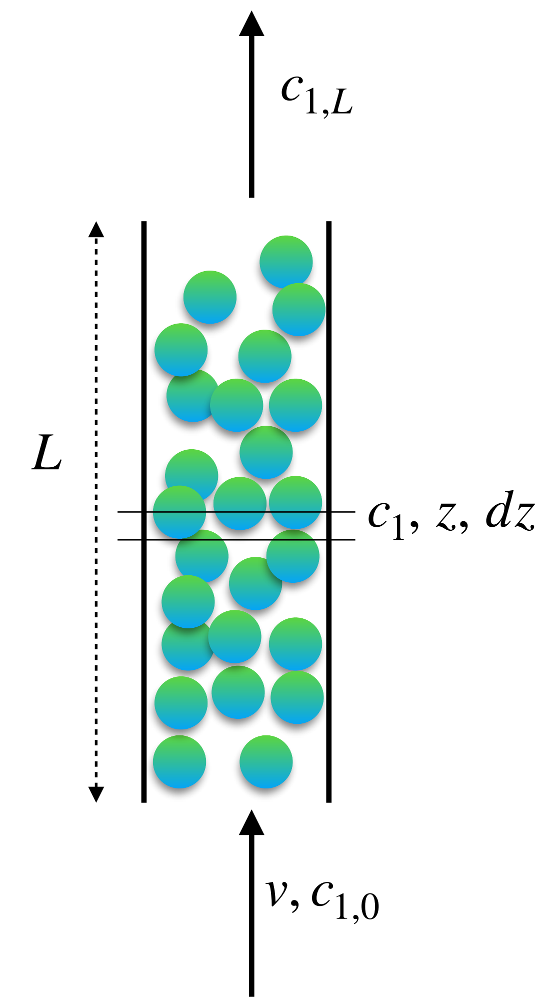
$$\Rightarrow \text{constant} = \ln c_{1,0}$$

$$\Rightarrow \ln\left(\frac{c_1}{c_{1,0}}\right) = -\frac{kaz}{v}$$

$$\Rightarrow ka = -\frac{v}{z}\ln\left(\frac{c_1}{c_{10}}\right)$$

$$\Rightarrow c_1 = c_{1,0}\exp\left(-\frac{kaz}{v}\right)$$

$$\Rightarrow ka = -\frac{v}{L}\ln\left(\frac{c_{1L}}{c_{10}}\right)$$



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Exercise problem 2
Overall mass transfer coefficient

A distillation column tray is contacting liquid benzene with its vapor. Bulk liquid and vapor mole fractions are 0.2 and 0.7, respectively. The equilibrium relationship, liquid-side and vapor-side mass transfer coefficients are given below.

Calculate the flux and interfacial mole fraction.

At interface, $y_{1i} = m x_{1i}$ $\quad m = 2 \quad k_x = 1 \dfrac{\text{mole}}{\text{m}^2\text{s}} \quad k_y = 1 \dfrac{\text{mole}}{\text{m}^2\text{s}}$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x} = 3 \qquad K_y = 0.33 \text{ mole m}^{-2}\text{ s}^{-1}$$

$$\mathbf{N_1 = K_y(y_1 - mx_1)} = 0.33(0.7 - 2 * 0.2) = 0.099 \text{ mole m}^{-2}\text{ s}^{-1}$$

$$N_1 = k_y(y_1 - y_{1i}) \qquad \Rightarrow y_{1i} = y_1 - \frac{N_1}{k_y} = 0.7 - \frac{0.099}{1} = 0.601 \qquad \Rightarrow x_{1i} = y_{1i}/m = 0.3$$

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